Basic AC Circuits

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AC Equations Presented

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$$a = A_m \sin \omega t$$
$$\omega = 2\pi f$$
$$f = \frac{1}{T}$$
$$a = A_m(\sin \omega t + \theta)$$
$$A_{avg} = \frac{2A_m}{\pi}$$
$$V_{RMS} = \frac{Vp}{\sqrt{2}}$$
$$\theta = \sin^{-1}\left(\frac{d_2}{d_1}\right)$$

Chapter 02

$$X_C = \frac{1}{2\pi f C}$$
$$X_L = 2\pi f L$$

Chapter 03

$$B_C = 2\pi f C$$
$$B_L = \frac{1}{2\pi f L}$$

Chapter 05

$p = VI \cos \phi\,(1 - \cos 2\omega t) + \ldots VI \sin \phi\,(\sin 2\omega t)$

$p_R = VI - VI \cos 2\omega t$

$q_L = VI \sin 2\omega t$

$q_C = -VI \sin 2\omega t$

$$W_R = \frac{VI}{f_P}$$
$$W_L = LI^2$$
$$W_C = CV^2$$

$P = VI \cos \phi$

$PF = \cos \phi$

Chapter 06

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$
$$Q_{coil} = \frac{Q_L}{P}$$
$$Q_{coil} = \frac{X_L}{R_S}$$
$$Q_S = \frac{X_L}{R + R_S} = \frac{X_C}{R + R_S}$$
$$V_X = Q_S V_g$$
$$BW = f_2 - f_1$$
$$BW = \frac{f_r}{Q}$$
$$f_1 = f_r - \frac{BW}{2}$$
$$f_2 = f_r + \frac{BW}{2}$$
$$f_1 = \frac{R}{4\pi L}\left(-1 + \sqrt{1 + \frac{4L}{R^2 C}}\right)$$
$$f_2 = \frac{R}{4\pi L}\left(1 + \sqrt{1 + \frac{4L}{R^2 C}}\right)$$
$$Q_S = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Chapter 07

$$Q_P = \frac{Q_L}{P}$$
$$Q_P = \frac{R}{X_L}$$
$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_S^2 C}{L}}$$
$$I_X = Q_P I_g$$
$$f_1 = \frac{1}{4\pi C R}\left(-1 + \sqrt{1 + \frac{4R^2 C}{L}}\right)$$
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Chapter 08

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$$s = j2\pi f$$
$$G(s)_{dB} = Av_{dB} = 20\log Av$$
$$G(s) = \frac{1}{1 + s\tau}$$
$$G(s) = 1 + s\tau$$
$$G(s) = \frac{1}{s\tau}$$
$$G(s) = s\tau$$
$$G(s) = K$$
$$X_L = sL$$
$$X_C = \frac{1}{sC}$$
$$S_m = \frac{dB_{HF} - dB_{LF}}{\log HF - \log LF}$$
$$S_p = \frac{deg_{HF} - deg_{LF}}{\log HF - \log LF}$$

Chapter 09

$$k = \frac{\phi_S}{\phi_P}$$
$$\eta = \frac{P_S}{P_P}$$
$$r = \frac{N_P}{N_S}$$
$$\frac{N_P}{N_S} = \frac{V_P}{V_S}$$
$$\frac{V_P}{V_S} = \frac{I_S}{I_P}$$
$$r = \sqrt{\frac{Z_P}{Z_L}}$$
$$r = \frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{I_S}{I_P} = \sqrt{\frac{Z_P}{Z_L}}$$

Chapter 1

Alternating Current

Chapter Starters

With this chapter, you have reached a significant milestone, the end of the study of direct current, and the beginning of the broader study of alternating current. Ohm's Law, however, has not changed. It applies equally to AC or DC circuits. It will serve you well as you venture into this new area of knowledge. So too will everything else you have learned up to now.

Alternating current is a gateway, but not just to series-parallel circuits employing inductors and capacitors as well as resistors. It is a gateway to a much larger realm of studies. Just like a computer which is quite a study by itself, but, when connected to Internet, becomes a window to a much larger world.

Alternating current is more than the availability of a larger energy source, although its significance as an energy source cannot be overstated. Alternating current is also the study of signals. Signals which allow us to communicate around the world or around the block. Signals which tell us how well our hearts are beating, or if we have a brain tumor. Signals which make our CD players produce music, and our TVs produce images. Signals may be classed as analog or digital. They may control our vehicle's speed or our DVD's rewind speed. They may do good things, such as allowing some people to hear clearly. They may do bad things, such as allowing some people to invade the privacy of others.

All of these examples are applications of alternating current. To study and learn these applications requires the student to have a thorough understanding of alternating current, what it is, how it works, how it is measured, and what it does. Your toolbox of knowledge has a lot in it right now and we need more space. Time to bring in another box of drawers called "AC," to fit underneath our "DC" toolbox. This and the following eight chapters will add quite a bit to your toolbox.

1.1 Generation of Alternating Current

Direct current may be defined as a current which flows in one direction only. **Alternating current** is a current which periodically changes its direction. Additionally, the **amplitude** or magnitude of the current constantly changes, in most cases. Chapter 9 in *Basic DC Circuits* provided explanation of how a conductor, forced to move through a magnetic field, may produce or generate an electric current. It is a relatively easy step to explain the generation of alternating current. The generation of direct current requires a conductor to move in a straight direction through a magnetic field. To generate alternating current, a conductor must move in a circular path through the magnetic field. A simplified drawing of how this may be accomplished is shown in Figure 1-1.

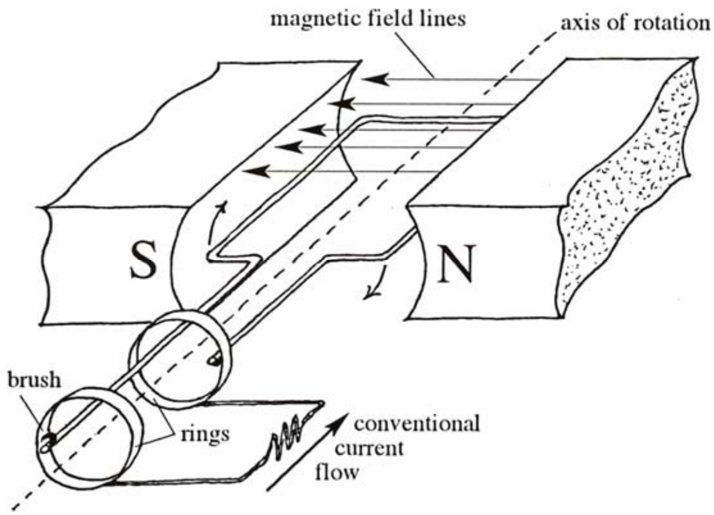

Figure 1-1. Alternating Current Generation

Chapter Objectives

This chapter will explain how an alternating current is generated using conventional magnetic theory from Chapter 9 in *Basic DC Circuits*. The mathematical characteristics used to describe AC will be fully illustrated, with special attention given to phase angles and their meaning. The origin and purpose of RMS labelling of voltages and currents will be clarified. This chapter will see the introduction of the oscilloscope as an AC measurement tool that allows us to see AC signals as well as to measure them. This chapter will end with basic instruction in AC troubleshooting and recognition of the tools now available to accomplish this task.

In Figure 1-1, a single loop of wire is shown as the conductor. The conductor loop is being forced to rotate about the axis as shown. Each end of the conductor is connected to its own ring through a conductive brush which makes continuous contact between the end of the conductor and the ring as the loop rotates. Attached to the rings is the external circuit through which the generated current will flow, producing power to a load. The loop must be forced to turn by the use of external mechanical power. In the case of a power generation station, fossil fuels or nuclear energy may be used to generate heat which then produces steam that is used to force generators to rotate, producing electrical energy. In a vehicle, the engine forces the rotation of a generator (a slightly different version called an alternator) via a mechanical belt coupling.

Consider the rotation of the loop of wire in its present position in Figure 1-1, using the right-hand generator rule. Recall that the thumb points in the direction of motion, the first finger points in the direction of magnetic flux, and the second finger will indicate the direction of conventional current flow. Application of this rule, to either side of the loop will confirm the indicated direction of current flow through the external circuit. As the loop continues to rotate, however, the situation becomes more complex. To explain what happens requires the view of Figure 1-2, which is the same setup as Figure 1-1, but viewed looking into the axis of rotation, from the ring end.

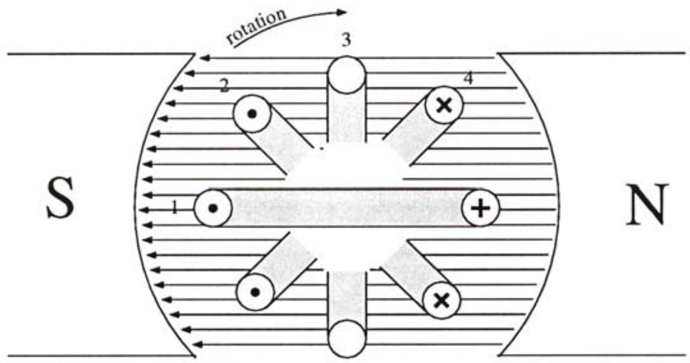

Figure 1-2. Generator Action

We will assume the loop is rotating at some constant speed. Recall Faraday's Law from Chapter 9 in *Basic DC Circuits*, which stated that the voltage produced across a conductor moving within a magnetic field is proportional to the rate at which flux lines are cut by the conductor. In position 1, both sides of the loop are moving vertically, cutting flux lines at the maximum possible rate. The voltage produced at this point is therefore maximum, and, assuming a completed circuit path, the current at this point is also at maximum.

As the loop rotates to position 2, the rate at which flux lines are cut diminishes because the motion of the conductor vertically is slower since part of the motion is horizontal. The voltage and current are thus lower than at position 1. When the conductor reaches position 3, it is moving parallel to the flux lines. No lines are being cut since vertical motion has ceased. At position 3 there is no voltage or current. Notice the direction of current flow in the left side of the loop, at positions 1 and 2. In both cases, current is flowing towards the viewer (the student should check this using the right-hand generator rule). At position 3, current stops. At position 4, this same side of the loop now begins cutting flux lines again, but it is now moving down instead of up. This means that the current direction in that part of the loop changes and now flows away from the viewer. The change in current direction also means the polarity of the voltage produced also reverses. If the voltage magnitude and polarity are plotted on a time scale, corresponding to the rotation of the loop, it will produce the voltage versus time plot illustrated in Figure 1-3. The voltage versus time plot is called a waveform. The waveform is continued for more than a complete revolution of the loop in order to illustrate the repetitive nature of this waveform.

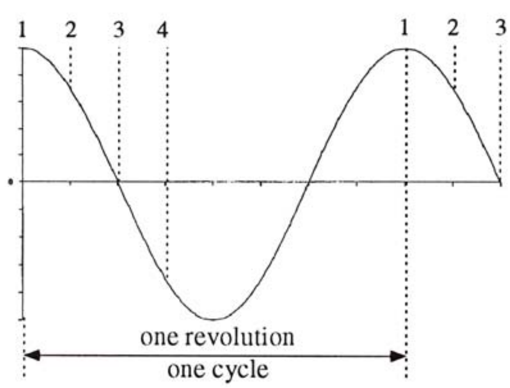

Figure 1-3. Voltage Waveform

When the loop has completed a full revolution, the voltage will again be at its maximum value shown at position 1 on the right side of Figure 1-3. The waveform then repeats itself as the loop begins another revolution. One complete revolution of the loop corresponds to what is called **one cycle** of the waveform. As long as the loop continues to turn at a constant speed, the waveform will keep repeating and is thus called a **repetitive** waveform, or it may be referred to as **periodic** in nature. An **aperiodic** waveform does not repeat itself at regular intervals.

It should be realized that if the rotating loop is connected to a completed circuit, the voltage waveform of Figure 1-3 will be applied across some external load resistance, resulting in a current waveform which will have exactly the same characteristics as the voltage waveform. The maximum amplitude of the current waveform will be equal to the maximum amplitude of the voltage waveform divided by the load resistance. Similarly, when the voltage is zero volts, at position 3, the current will be zero amperes.

Notice the shape of the voltage waveform. It happens that the rate at which the flux lines are cut by circular motion of a conductor corresponds to a mathematical function called a **sinusoid**. The waveform is therefore referred to as a **sinusoidal** waveform, or, more simply, a **sine wave**. One loop of wire rotating even in a strong magnetic field would not be able to produce much current. The usefulness of a generator is increased by having a very large number of loops rotating at the same time within a magnetic field made stronger by the employment of **field windings**, or additional coils of wire through which a current is applied to create an intense magnetic field. Neither explanations or illustrations of generators or motors is within the scope of this text. They are complex applications of magnetic theory, requiring separate study. The simple generator of Figure 1-1 was presented as a means to explain the creation of a sine wave. Most electronics curriculums include the study of generators and motors somewhere, but seldom in an introductory course. The next section will illustrate the characteristics of the sine wave in more detail.

1.2 Characteristics of AC Waveforms

The sine wave is described by Equation 1-1, presented in general terms, but equally applicable to voltage waveforms or current waveforms.

Equation 1-1 $$a = A_m \sin \omega t$$

In Equation 1-1, a is the instantaneous value of the waveform at time t . A_m is the peak amplitude of the waveform, ω is the angular velocity of the waveform, and "sin" is the sinusoidal mathematical function. Angular velocity requires considerable explanation, but first, some basics are presented in Figure 1-4 to illustrate the instantaneous amplitude.

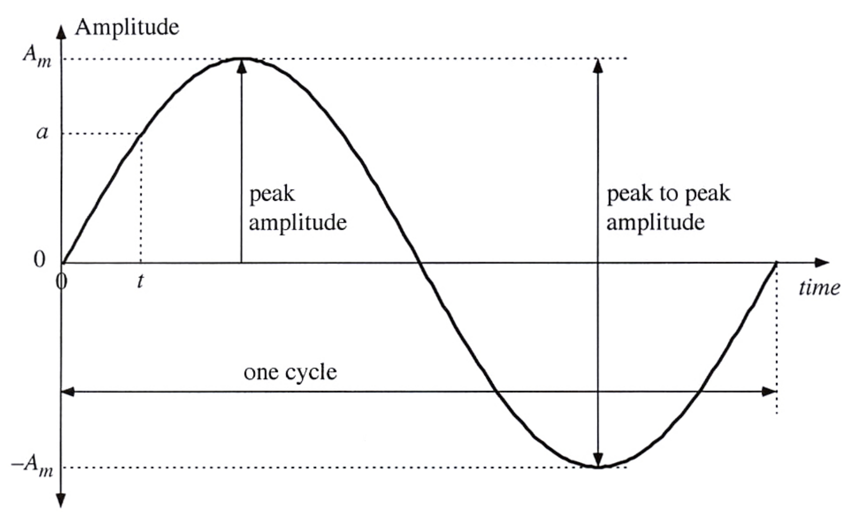

Figure 1-4. Instantaneous Amplitude

Voltage and Current Values

The first half of the waveform of Figure 1-4, assuming the waveform is centered vertically at zero amplitude, is referred to as the **positive half** of the waveform, the rest being the **negative half**. The measurement from zero amplitude to either the positive peak or the negative peak is called the **peak** amplitude. The measurement from the positive peak to the negative peak, regardless of where the zero amplitude level is located, is called the **peak-to-peak** amplitude. When the waveform represents voltage or current, an instantaneous value would be recorded, for example, as 18.3 V or 846 µA, however peak amplitudes must be labeled as 10.3 Vp or 3.58 mAp, while peak-to-peak amplitudes would be correctly printed as 625 mVpp or 115 µApp. The proper use of the "p" or "pp" after the unit symbol is essential to clarify the meaning of the amplitude in question, although for simplicity we will not subscript them, but you may do that if you wish, so 10.3 Vp or 10.3 $\mathrm{V_p}$ is acceptable as is 115 µApp or 115 $\mathrm{\mu A_{pp}}$. Since these are unit symbols, they are never italicized.

When used to represent a voltage or current, Equation 1-1 would be written as $v = V_m \sin \omega t$ or $i = I_m \sin \omega t$. Note that mathematical functions such as sin, cos, tan and so forth are not italicized. Using numbers in place of the peak values represented by V_m and I_m, would result in, for example, $v = 180$ mV $\sin \omega t$ or $i = 420$ µA $\sin \omega t$. It is also interesting to note that the values used in these examples are actually 180 mVp and 420 µAp, since they are the peak values of the waveform, but within the sinusoidal expression they are correctly written just as shown. It would not be incorrect to write $v = 180$ mVp $\sin \omega t$ or $i = 420$ µAp $\sin \omega t$, it is just that nobody writes it that way. As a student, you just have to remember that the value in the sinusoidal expression is always the peak value of the waveform. These two waveforms are illustrated in Figure 1-5.

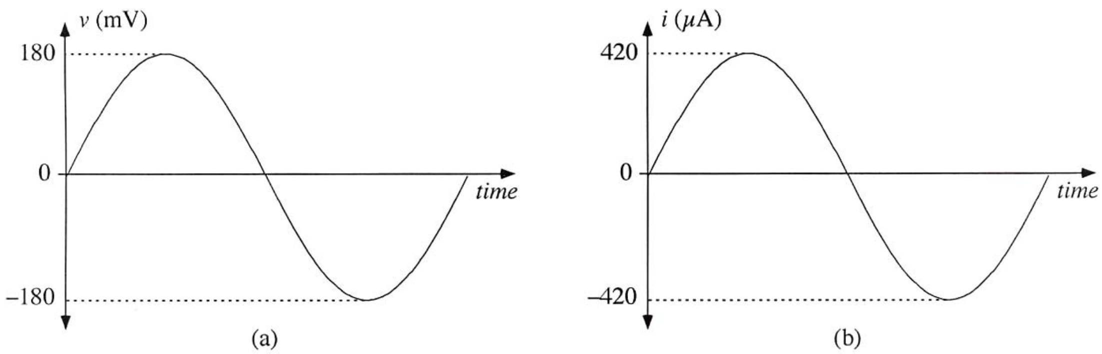

Figure 1-5. Voltage and Current Waveforms

The amplitudes of the waveforms of Figure 1-5 could be referred to as 180 mVp, 360 mVpp, 420 µAp or 840 µApp. Later in this chapter we will see that there are still other ways of expressing voltage or current amplitudes.

Time Values

The time required to complete one cycle of the waveform is equal to the time needed for the loop of wire in Figure 1-1 to complete one revolution. There are a number of ways of expressing this time. Since one revolution is one complete circle, that circle can be measured in units of **degrees** or **radians**, and such measurement is called **angular displacement** or **angular distance**. One complete revolution, or one circle, is equal to 360 degrees, and is also equal to 2π radians, or 6.28 radians, abbreviated 6.28 rad without an "s" at the end. Angular displacement is usually illustrated on an X-Y graph as a line rotating counter-clockwise with zero located at its east point, as shown in Figure 1-6.

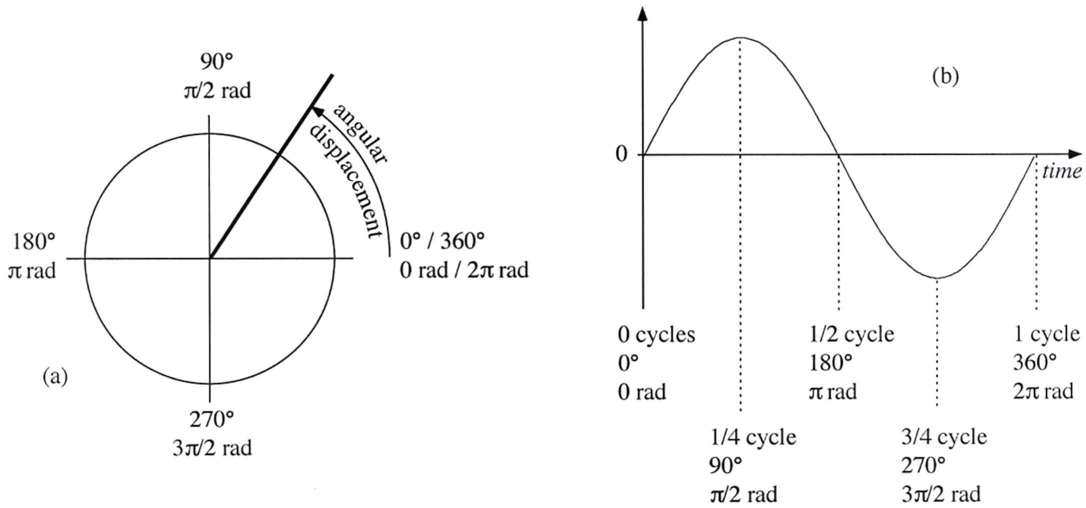

Figure 1-6. Angular Displacement

The distance the line moves from the 0° axis is the angular displacement, or ωt in the sinusoidal expression. That angular distance or displacement is the product of ω and t. Omega, or ω, is the **angular velocity** of the line, and is always expressed in units of radians per second, abbreviated rad/s. Time, t, is the measure of whatever time is required for the generator loop to move through the angular displacement. If, for example, the loop is moving at an angular velocity of 6.28 rad/s, for a time of 1.0 s, the angular displacement would be 6.28 rad/s × 1.0 s, or 6.28 rad, which is exactly one revolution or one cycle. As another example, a loop rotating at a speed of 825 rad/s for a time of 1.8 ms, will rotate a distance of 1.49 rad, which is just less than one quarter of a cycle, or 90°.

It is often necessary to convert between angular measurement in degrees or radians. The conversion equality is 180° = π rad, so, as with other conversions, it can be written as

$$\frac{180°}{\pi \text{ rad}} \text{ or } \frac{\pi \text{ rad}}{180°}$$

Contained within ω is information pertaining to the **period** of the waveform, which is the time taken to complete one cycle, and the **frequency** of the waveform, which is the number of cycles of the waveform occurring in one second. These quantities are defined in Equations 1-2 and 1-3.

Equation 1-2
$$\omega = 2\pi f$$

Equation 1-3
$$f = \frac{1}{T}$$

Frequency, represented by f in Equation 1-2 was once expressed in units of cycles per second. The 2π in Equation 1-2 represents 2π radians per cycle, thus the units of this equation were radians per cycle times cycles per second, which reduces to units of radians per second, the normal units of ω. Frequency is no longer measured in these units, but is now expressed in units of Hertz, abbreviated Hz, and named after Heinrich R. Hertz (1857-1894), a German physicist. One Hertz is equal to one cycle per second. In Equation 1-3, T represents period and is measured in seconds. With older units, frequency was in cycles per second, and its reciprocal, T, was thus seconds per cycle. Now, T is measured in seconds and it is understood that the measurement extends for one whole cycle. T, in seconds is the reciprocal of f in Hertz. Thus, a frequency of 50 kHz has a period of 20 μs. Several examples are provided to illustrate typical uses of Equations 1-1, 1-2 and 1-3.

Example 1-1

If the period of a 35 Vp waveform is 15.8 µs, determine f and ω and write the sinusoidal expression for the waveform.

$f = 1 / T = 1 / 15.8$ µs = 63.3 kHz
$\omega = 2\pi f = 2 \times \pi \times 63.3$ kHz = 398 krad/s
$v = 17.5$ V sin 398 kt

When substituting into Equation 1-1 to find an instantaneous value, it is important to be sure that your calculator is set to the correct angular mode. To evaluate the sine of ωt, when ωt is in radians, the calculator must be in radian mode. If the calculator is set to degree mode, then ωt must first be converted into degrees.

Example 1-2

Determine the instantaneous voltage at $t = 820$ µs if $v = 15$ V sin 3 kt.

With the calculator in radian mode:
$v = 15$ V sin (3 krad/s × 820 µs) = 15 V sin 2.46 rad = 15 V × 0.630 = 9.45 V

With the calculator in degree mode:
$v = 15$ V sin (3 krad/s × 820 µs × 180° / π rad) = 15 V sin 141° = 15 V × 0.630 = 9.45 V

Example 1-3

If $i = 800$ µA sin ωt, and the period of the waveform is 120 µs, find i at $t = 70$ µs and show where this instantaneous current is on a sketch of the waveform.

$T = 120$ µs
$f = 1 / T = 1 / 120$ µs = 8.33 kHz
$\omega = 2\pi f = 2 \times \pi \times 8.33$ kHz = 52.4 krad/s
$i = 800$ µA sin 52.4 kt = 800 µA sin (52.4×10^3 rad/s × 70 µs)
$i = 800$ µA sin (3.67 rad) = 800 µA sin (3.67 rad × 180° / π rad) = 800 µA sin 210°
$i = 800$ µA × –0.5 = –400 µA

Note that sin 210° = sin 330° = –0.5, so one must determine which level of –400 µA corresponds to a time of 70 µs. Figure 1-7 illustrates this example.

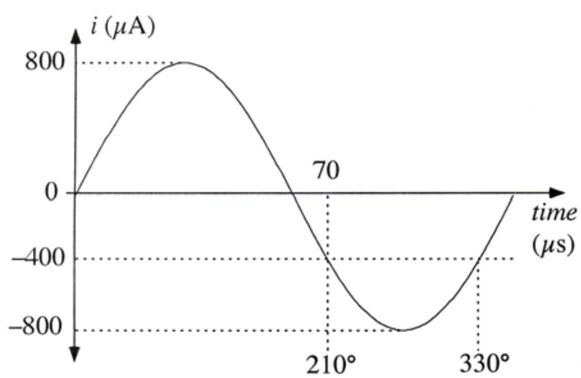

Figure 1-7. Illustration for Example 1-3

Example 1-4

Given that v = 450 mV sin 800 t, determine (a) the peak voltage, (b) the peak-to-peak voltage, (c) the voltage at t = 1.0 ms, (d) the voltage at t = 5.0 ms, (e) the frequency of the waveform, and (f) the period of the waveform.

(a) the peak voltage = 450 mVp
(b) the peak-to-peak voltage = 2 × Vp = 900 mVpp
(c) v = 450 mV sin (800 rad/s × 1.0 ms × 180° / π rad) = 450 mV sin 45.8° = 323 mV
(d) v = 450 mV sin (800 rad/s × 5.0 ms × 180° / π rad) = 450 mV sin 229° = –341 mV
(e) $f = \omega / 2\pi$ = 800 rad/s / 2π = 127 Hz
(f) $T = 1 / f$ = 1 / 127 Hz = 7.85 ms

Example 1-5

If the instantaneous value of current at 155° of the cycle is 3.91 mA, and t at this same point of the cycle is 247 µs, determine (a) the peak value of the current, (b) the period of the current, and (c) the frequency of the current.

(a) 3.91 mA = I_m sin 155°
I_m = 3.91 mA / sin 155° = 3.91 mA / 0.423 = 9.25 mAp
(b) If 155° corresponds to 247 µs, then 360° will correspond to T

$$\frac{155°}{247\,\mu\text{s}} = \frac{360°}{T}$$

T = 247 µs × 360° / 155° = 574 µs
(c) $f = 1 / T$ = 1 / 574 µs = 1.74 kHz

Example 1-6

If the instantaneous value of a 10 kHz waveform is 26.5 V and the waveform is 85 Vpp, what two times could the waveform be at this voltage level?

The peak value is 85 Vpp / 2 = 42.5 Vp
26.5 V = 42.5 V sin ωt
sin ωt = 26.5 V / 42.5 V = 0.624
sin ωt = 0.624
Evaluate the arcsine or inverse sine of both sides of the equation with the calculator in degree mode
$\sin^{-1} (\sin \omega t) = \sin^{-1} (0.624)$
$\omega t = \sin^{-1} 0.624 = 38.6°$
This level will also occur at 180° – 38.6°, or 141°
$T = 1 / f$ = 1 / 10 kHz = 100 µs
one time occurs 38.6° into the cycle, or 38.6° / 360° × 100 µs = 10.7 µs
the other time occurs 141° into the cycle, or 141° / 360° × 100 µs = 39.3 µs
The two times are 10.7 µs and 39.3 µs.